# Introduction to Adaptive Volumetric Efficiency


## Abstract

Development of advanced engine control systems for the modern four-stroke gasoline and diesel internal combustion engines (ICE) are being driven by:

- Demand for maximum fuel economy and thermal efficiency
- Increasingly stringent exhaust emission standards (ULEV, SULEV, PZEV)
- Development of advanced actuators that alter fundamental physical properties of the engine in a very large hyperspace
- Hybrid controls management of multiple power sources
- Flexible-fuel controls management for variable fuel vehicles
- Consumer demand for improved quality and performance from smaller displacement engines
- Comprehensive On-Board-Diagnostics (OBD2) requirements for increasing service duration

The complexity of modern powertrain controls can be quantified by the number of calibration parameters in the controller. Powertrain controllers with **over fifteen-thousand** calibration parameters are already a reality (*Toyota SAE-2004-21-0063, dSpace "Challenges to a Modern ECU Calibration System"*).

The root cause of the increased complexity can be traced to the cylinder. Because there are no production quality sensors for measuring cylinder mass flows, pressure and temperature, it is common practice to use physical modeling of varying complexity to infer cylinder state variables from available measurements. The quintessential model at the heart of all powertrain controllers is the multi-variable nonlinear volumetric efficiency calculation that describes the engine pumping performance. Volumetric efficiency is used to determine the cylinder states for engine load that is utilized throughout the entire control strategy. Having an accurate volumetric efficiency model under all operating conditions is essential for high performance emission and drivability quality.

The objective of this paper is to introduce a new adaptive high-fidelity volumetric efficiency physical model that can be employed as the foundation for a precision powertrain real-time control system. With this control, it will be possible to significantly reduce calibration expenditures while improving performance, quality and robustness.

## Introduction

Volumetric efficiency varies in 19-dimensional and expanding hyperspace. If we collect 10 pieces of data at each hyperpoint at quasi-steady state conditions averaging 1 minute at each, we would have $10^{19}$ number of points taking $4.5e^{12}$ years to complete! Obviously we need to design the experiment to sample the most useful hyperpoints that generates the highest quality model while minimizing expenditures. To accomplish this we need a high-fidelity physical model that closely fits the hyperplane with a few tuning parameters to match actual engine volumetric efficiency.

Phenomenological factors that we know that influence volumetric efficiency are:

1. Cylinder charge temperature
2. Cylinder residual mass ratio
3. Exhaust gas recirculation (EGR)
4. Variable cam timing
5. Variable cam lift
6. Variable intake manifold volumes and runner length controls
7. Variable intake manifold runner air velocity controls
8. Cylinder valve deactivation controls
9. Charge heating in manifold and cylinder from the walls(low speed)
10. Backflow (valve timing at low flow and speed)
11. Tuning (middle speed range valve and manifold torque tuning)
12. Choking and flow friction (higher speeds)
13. Ram effects (higher speeds)
14. Fuel vapor (different for each fuel type and moves with commanded fuel/air ratio)
15. Fuel endothermic evaporations resulting in temperature drops
16. Water vapor (moves with relative humidity)
17. Air filter degradation

Incredibly, one of the most important factors for determining volumetric efficiency is the cylinder charge temperature and is missing from most modern control systems. Cylinder charge temperature stationary effects are weakly provided for in calibration look-up tables and the dynamic effects are completely ignored. Consequently, calibrators are perpetually changing their look-up table values until an acceptable error is achieved (or they run out of time on the project). The next most important factor is the residual mass ratio that changes in the following hyperspace:

1. Compression ratio
2. Engine Speed
3. EGR
4. Purge
5. Fuel stoichiometric mass
6. Fuel to air ratio
7. Ambient pressure
8. Intake manifold pressure
9. Intake manifold temperature
10. Exhaust manifold pressure
11. Exhaust manifold temperature
12. Engine wall temperature
13. Engine combustion chamber head/valve temperatures
14. Intake cam-angle
15. Intake cam-lift
16. Exhaust cam angle
17. Exhaust cam lift
18. Intake volume controls
19. Intake runner area controls

This paper is organized as follows

- Section-1: Classical Volumetric Efficiency Definition
- Section-2: Heywood Physical Model
- Section-3: Servati Hybrid Physical-Regression Model
- Section-4: Ford Affine Pressure Regression Model by Messih
- Section-5: Andersson Physical Molar Volume Extension Model
- Section-6: Variable Cam Timing Molar Volume Extension
- Section-7: 10-Dimensional Energy Balance Molar Volume Hybrid
- Section-8: Cylinder Residual Mass Adaptive Observer

**Nomenclature**

| Parameter: | Definition: | Units: |
|---|---|---|
| $\eta_{ca}$ | Cylinder volumetric efficiency of combustible mass air charge | - |
| $Mc$ | Cylinder mean total mass charge | mg |
| $Mca$ | Cylinder mean air mass charge | mg |
| $Mcas$ | Standard cylinder air mass charge | mg |
| $\mathrm{M}ca$ | Cylinder mean air mass moles | moles |
| $Mcf$ | Cylinder mean fuel mass charge | mg |
| $Mcr$ | Cylinder mean residual mass charge | mg |
| $\Delta Mcr$ | Cylinder mean residual mass charge offset | mg |
| $Mcw$ | Cylinder mean water vapor mass charge | mg |
| $Wc$ | Cylinder mean total mass flow | g/s |
| $Wca$ | Cylinder mean air mass flow | g/s |
| $Wcas$ | Standard cylinder air mass flow | g/s |
| $Wcf$ | Cylinder mean fuel mass flow | g/s |
| $Wcr$ | Cylinder mean residual mass flow | g/s |
| $\Delta Wcr$ | Cylinder mean residual mass flow offset | g/s |
| $Wcw$ | Cylinder mean water vapor mass flow | g/s |
| $Pa$ | Ambient mean air absolute pressure | KPa |
| $Pas$ | Standard ambient air absolute pressure | KPa |
| $Pe$ | Exhaust-manifold mean total absolute pressure | KPa |
| $Pc$ | Cylinder mean total absolute pressure | KPa |
| $Pt$ | Throttle mean absolute pressure just upstream of valve | KPa |
| $Pi$ | Intake-manifold mean total absolute pressure | KPa |
| $Pia$ | Intake-manifold mean partial pressure of combustible air | KPa |
| $Pif$ | Intake-manifold mean partial pressure of fuel vapor | KPa |
| $Pir$ | Intake-manifold mean partial pressure of residual gas | KPa |
| $Piw$ | Intake-manifold mean partial pressure of water vapor | KPa |
| $Ta$ | Ambient mean air absolute temperature | K |
| $Tas$ | Standard ambient air absolute temperature | K |
| $Tw$ | Engine wall mean absolute temperature | K |
| $Tws$ | Standard engine wall temperature | K |
| $Te$ | Exhaust-manifold gas mean absolute temperature directly after EVO at valve | K |
| $Tc$ | Cylinder mean charge temperature directly after IVC | K |
| $Ti$ | Intake-manifold mean charge temperature | K |
| $Ne$ | Mean engine crankshaft speed | Rev/s |
| $\varphi$ | Mean fuel / air equivalence ratio | mg/mg |
| $\varphi s$ | Standard fuel / air equivalence ratio | mg/mg |
| $aCe$ | Exhaust valve opening angle (EVO) | CAô |
| $aCi$ | Intake valve opening angle (IVO) | CAô |
| $r_c$ | Engine compression ratio | cc/cc |
| $x_r$ | Mean cylinder residual mass ratio | mg/mg |
| $Cv$ | Specific heat at constant volume | |
| $Cp$ | Specific heat at constant pressure | |

| | | |
|---|---|---|
| $\gamma$ | Ratio of specific heats $\gamma = \frac{Cp}{Cv} = \frac{Cv + R}{Cv}$ where $1 \le \gamma \le 2$ | - |
| $R$ | Universal gas ideal constant | (KPa L)/ (K mg) |
| $TDC$ | Piston at top-dead-center in the cylinder | - |
| $BDC$ | Piston at bottom-dead-center in the cylinder | - |
| $EVO$ | Exhaust valve open crank angle | CAô |
| $EVC$ | Exhaust valve closed crank angle | CAô |
| $IVO$ | Intake valve open crank angle | CAô |
| $IVC$ | Intake valve closed crank angle | CAô |
| $Vc$ | Cylinder clearance volume | L |
| $Vd$ | Cylinder swept volume | L |
| $Vr$ | Cylinder residual molar volume at EVC | L |
| $Vri$ | Cylinder residual molar volume at IVC | L |
| $Vaf$ | Cylinder air fuel charge molar volume at IVC | L |
| $Va$ | Cylinder air charge molar volume at IVC | L |
| $Vf$ | Cylinder fuel charge molar volume at IVC | L |
| $\Delta Ve$ | Cylinder volume change with VCT at EVC | L |
| $\Delta Vi$ | Cylinder volume change with VCT at IVC | L |

## 1 Volumetric Efficiency Classical Definition

Cylinder air mass charge definition

(0.1) $$\eta_{ca} \equiv \frac{\text{Actual } \mathbf{Combustible} \text{ Cylinder Air Mass Charge}}{\text{Theoretical } \mathbf{Combustible} \text{ CylinderAir Mass Charge at Stationary Upstream Conditions}}$$

Cylinder air mass flow equivalent definition

(0.2) $$\eta_{ca} = \frac{\text{Actual } \mathbf{Combustible} \text{ Cylinder Air Mass Flow}}{\text{Theoretical } \mathbf{Combustible} \text{ Cylinder Air Mass Flow at Stationary Upstream Conditions}}$$

Defined across the cylinder from the intake manifold conditions (lumped approach assuming port conditions match the intake-manifold reservoir)

(0.3) $$\eta_{ca} \equiv \frac{Mca}{\left(\frac{Pia}{R \cdot Ti}\right) \cdot Vd} = \frac{R \cdot Ti \cdot Mca}{Pia \cdot Vd}$$

Alternative air mass flow definition

(0.4) $$\eta_{ca} \equiv \frac{2 \cdot Wca}{\left(\frac{Pia}{R \cdot Ti}\right) \cdot Vd \cdot Ne} = \frac{2 \cdot R \cdot Ti \cdot Wca}{Pia \cdot Vd \cdot Ne}$$

Or defined across the throttle for the entire engine assuming the upstream throttle conditions match ambient:

(0.5) $$\eta_{ca} \equiv \frac{Mca}{\left(\frac{Pa}{R \cdot Ta}\right) \cdot Vd} = \frac{R \cdot Ta \cdot Mca}{Pa \cdot Vd}$$

And finally across the throttle based upon air mass flow

(0.6) $$\eta_{ca} \equiv \frac{2 \cdot Wca}{\left(\frac{Pa}{R \cdot Ta}\right) \cdot Vd \cdot Ne} = \frac{2 \cdot R \cdot Ta \cdot Wca}{Pa \cdot Vd \cdot Ne}$$

Note that $Pia$ is the partial pressure of combustible air in the manifold which cannot be measured directly because of residuals and fuel vapor inside the manifold, therefore most all $\eta_{ca}$ definitions substitute $Pia = Pi$ using the total intake manifold pressure instead.

(0.7) $$Mc = Mca + Mcf + Mcr + Mcw$$

(0.8) $$Pi = Pia + Pif + Pir + Piw$$

## 2 Volumetric Efficiency Model-1: Heywood Physics Model

Heywood "Internal Combustion Engine Fundamentals", 1988 p210, (6.2)

(1.1)
$$\eta_{ca} = \left(\frac{\mathrm{M}ca}{\mathrm{M}a}\right)\cdot\left(\frac{Pi}{Pa}\right)\cdot\left(\frac{Ta}{Ti}\right)\cdot\left(\frac{1}{1+\frac{\varphi}{\varphi s}}\right)\cdot\left\{\left(\frac{r_c}{r_c-1}\right)-\left(\frac{1}{\gamma\cdot(r_c-1)}\right)\cdot\left[\left(\frac{Pe}{Pi}\right)+(\gamma-1)\right]\right\}$$

$\mathrm{M}ca$ is the moles of combustible air in the cylinder at BDC-IVC and $\mathrm{M}a$ is the moles of combustible air at upstream conditions.

One-dimensional isentropic steady compressible flow temperature correction factor (deviations from standard mapping temperature)

(1.2)
$$Cta = \left(\frac{Tas}{Ta}\right)^{\frac{1}{2}}$$

Charge heating in manifold and cylinder (deviation from standard mapping engine coolant/wall temperature)

(1.3)
$$Ctw = F\left(\frac{Tws}{Tw}\right)$$

Altitude variations from standard mapping conditions $(\leq 0.03)$

(1.4)
$$Cpa = \frac{Pa}{Pas}$$

Alternatively including water vapor pressure:

(1.5)
$$Cpaw = \frac{Pa-Pw}{Pas-Pws}$$

Partial pressure of fuel and water vapor from standard mapping conditions (fuel/air deviations $\leq 0.02$ with isooctane)

(1.6)
$$Cvfw = \frac{1}{1+\frac{\varphi}{\varphi s}\cdot\frac{\mathrm{M}a}{\mathrm{M}f}+\frac{\varphi w}{\varphi ws}\cdot\frac{\mathrm{M}a}{\mathrm{M}w}} = \frac{1}{1+\frac{\varphi}{\varphi s}\cdot\frac{28.966}{114.23(C_8H_{18})}+\frac{\varphi w}{\varphi ws}\cdot\frac{28.966}{18.02}}$$

## 3 Volumetric Efficiency Model-2: Servati Hybrid Physical-Regression

Servati and DeLosh SAE-86-0328

(2.1) $$\eta_{ca} = \alpha_0 + \alpha_1 T_i^{0.5} + \alpha_2 \left(P_e/P_i\right) + \alpha_3 \left(T_i/T_e\right) + \alpha_4 \left(N_e/T_i\right)^{0.8} + \alpha_5 \left(\left(N_e^2/T_i\right)\left(\left(T_i/T_e\right) + \left(r_c - 1\right)\right)\right)$$

As stated by the authors, this regression possesses a functional form relating to physical processes within the engine:

- Term-1 relates the inlet valve Mach index which is proportional to the ratio of characteristics gas velocity at the inlet valve to inlet sonic velocity
- Term-2 relates the flow characteristics through the inlet valve for a considerable portion of the intake stroke
- Term-3 relates to the residual gas dynamics (prominent at low speed, low load and idle)
- Term-4 relates turbulent flow heat transfer taking place from cylinder walls to the air entering the cylinder
- Term-5 relates increasing inlet gas velocity fluid friction in the intake port

## 4 Volumetric Efficiency Model-3: Ford Affine Pressure Regression

Ford Motor Company Model by Isis Messih Patent 5331936, 1994

The following regression has achieved an overall correlation $r = 0.98$ with mapping datasets produced from mean-value components for a production powertrain system with variable intake cam timing at stationary conditions based upon work at Ford Motor Company in the early 90ôs.

(3.1) $$y = a \cdot x + b$$

(3.2) $$Pi = Fps \cdot Mca + Fpo \cdot \frac{Pa}{Pas}$$

(3.3) $$Fps(Ne) = a0 + a1 \cdot Ne + a2 \cdot Ne^2$$

(3.4) $$Fpo(Ne) = b0 + b1 \cdot Ne + b2 \cdot Ne^2$$

Or including intake cam timing into the power regression

(3.5) $$Fps(Ne, aCi) = a0 + a1 \cdot Ne + a2 \cdot Ne^2 + a3 \cdot aCi + a4 \cdot aCi^2 + a5 \cdot Ne \cdot aCi$$

(3.6) $$Fpo(Ne, aCi) = b0 + b1 \cdot Ne + b2 \cdot Ne^2 + b3 \cdot aCi + b4 \cdot aCi^2 + b5 \cdot Ne \cdot aCi$$

(3.7) $$\eta_{ca} = Fc(Tw, Ti) \cdot \frac{Mca}{Pi \cdot Vcd}$$

Where the temperature compensation $Fc$ has been derived to include both intake temperature and engine coolant heat transfer effects (includes $R$ ). Altitude compensation is only applied to the offset term $Fpo$ .

By utilizing the affine structure and having excellent correlation, the slope describes only the combustible product

(3.8) $$Fps = \frac{Pi}{Mca}$$

While the offset yields insight into the partial pressure of the residuals

(3.9) $$Fpo = \frac{Pas}{Pa} \cdot \left( Pif + Pir + Piw \right)$$

This model of volumetric efficiency is employed dynamically (recursive) in a mass airflow sensor based cylinder mass air charge state equation discretized in the angle domain sampled-averaged ever 45 CAộ and integrated every mean-cylinder intake event. Importantly, a predictor is formed where this state equation is integrated into the future such that fuel may be scheduled before IVO properly:

(3.10) $$Mca(k) = K(k) \cdot F + \left( \left( \frac{K(k)}{K(k-1)} \right) \cdot \left( 1 - K(k) \right) \right) \cdot Mca(k-1)$$

This assumes that the volumetric efficiency changes an insignificant amount between cylinder events since the current cylinder air mass charge is computed with the previously computed volumetric efficiency.

The accuracy of the model has shown excellent fuel/air control combined with the Aquino wall-wetting model and numerous feedforward devices for correction (including throttle-AE). From PZEV calibrators in 2004, it is claimed to have achieved 0.1 air/fuel control for the Ford Focus from cold start.

This regression model has the following concerns:

1. Missing charge temperature compensation (exhaust gas pressure and temperature deviations)
2. Missing EGR modeling (offset errors)
3. Missing air/fuel ratio modeling (both mass and evaporation factors)
4. Lumped temperature correction factors missing dynamic components
5. Missing isentropic/polytropic pressure ratio correction in slope term
6. Has known stationary time-varying offset errors (~10%)
7. High cost of system identification expenditures

## 5 Volumetric Efficiency Model-4: Andersson $\varphi, Pe$ Physical Molar Volume Extension

Linköping University Sweden, PhD Thesis #989 ñAir Charge Estimation in Turbocharged Spark Ignition Enginesò

The Andersson volumetric efficiency with correction factors for fuel/air equivalence ratio and exhaust pressure is derived using the molar form of the ideal gas law instead of the energy balance as in (Taylor, 1994, p510). The mass air charge is computed from deriving the combustible volume of air and fuel at the BDC-IVC intake stroke

(4.1)
$$Mca = \left(\frac{Pi}{Ri \cdot Ti}\right) \cdot Va$$

The molar form of the ideal gas law is

(4.2)
$$V = \frac{n \cdot R \cdot T}{P} = \left(n_1 + \ldots + n_n\right) \cdot \left(\frac{R \cdot T}{P}\right)$$
$$V_1 + \cdots + V_n = \left(n_1 + \ldots + n_n\right) \cdot \left(\frac{R \cdot T}{P}\right)$$
$$V_i = n_i \cdot \left(\frac{R \cdot T}{P}\right) \quad 1 \le i \le n$$

With the molar volumes at BDC-IVC intake stroke are divided as

(4.3)
$$Vaf = Va + Vf = Vd + Vc - Vr$$

To compute the expanded residual volume, we first start at the end of the exhaust-stroke at TDC-EVC. Knowledge of the cylinder residual mass is obtained by assuming that cylinder states equilibrate to that of the exhaust manifold reservoir:

(4.4)
$$Mcr = \left(\frac{Pc}{Rc \cdot Tc}\right) \cdot Vc \approx \left(\frac{Pe}{Re \cdot Te}\right) \cdot Vc$$

Assuming either an isentropic or polytropic process moving the piston down with the intake valve open to the intake-manifold reservoir and that the cylinder pressure equilibrates to the intake manifold pressure, we can derive

(4.5)
$$\left(\frac{Pi}{Pe}\right) = \left(\frac{Vc}{Vr}\right)^{\gamma} = \left(\frac{Tc}{Te}\right)^{\frac{\gamma}{\gamma - 1}}$$

The BDC residual mass volume can be calculated as

(4.6)
$$Vr = \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot Vc = \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \frac{Vd}{r_c - 1}$$

Where the clearance volume has been converted with compression ratio and displacement volume

(4.7)
$$r_c = \frac{\text{Maximum Cylinder Volume}}{\text{Minimum Cylinder Volume}} = \frac{Vd + Vc}{Vc}$$
$$Vc = \frac{Vd}{r_c - 1}$$
$$Vd + Vc = Vd \cdot \left(\frac{r_c}{r_c - 1}\right)$$

The cylinder volume available for the combined air fuel charge is

(4.8) $$Vaf = Vd + Vc - Vr = Vd \cdot \left(\frac{r_c}{r_c - 1}\right) - \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \frac{Vd}{r_c - 1} = Vd \cdot \left(\frac{r_c - \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}}}{r_c - 1}\right)$$

Using the control law (at stationary conditions assuming no fueling errors)

(4.9) $$\frac{\varphi}{\varphi s} = \frac{Mcf}{Mca}$$

The volume of inducted combustible air is:

(4.10) $$Va = \left(\frac{1}{1 + \frac{\varphi}{\varphi s}}\right) \cdot Vaf$$

To describe the actual pumping capabilities of the engine, Andersson borrows an expression for charge cooling based upon fuel vaporization from Hendricks (1996) and adds a linear tuning correction

(4.11) $$Mca = \frac{Pi \cdot Va}{Ri \cdot Tc} = Fca \cdot \left(\frac{Pi \cdot Vd}{Ri \cdot \left(Ti - Ct \cdot \left(\varphi^2 - 1\right)\right)}\right) \cdot \left(\frac{1}{1 + \frac{\varphi}{\varphi s}}\right) \cdot \left(\frac{r_c - \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}}}{r_c - 1}\right)$$

(4.12) $$Tc = Ti - Ct \cdot \left(\varphi^2 - 1\right)$$

The linear volumetric efficiency definition extended for $\{\varphi, Pe\}$ becomes

(4.13) $$\eta_{ca} = \frac{Fca}{Fct(Ti,\varphi)} \cdot \left(\frac{1}{1 + \frac{\varphi}{\varphi s}}\right) \cdot \left(\frac{r_c - \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}}}{r_c - 1}\right)$$

$$Fct(Ti,\varphi) = \frac{Ti - Ct \cdot \left(\varphi^2 - 1\right)}{Ti}$$

Power regression may be applied to $Fca, Fct$ and $Pe$ resulting in 3% accuracy at part-load claimed by the author.

Andersson then uses this stationary volumetric efficiency model in a dynamic manifold pressure model and combines it with an observer using an air mass offset state and applying the manifold pressure residuals

(4.14) $$\frac{d\hat{Pi}}{dt} = Kim \cdot \left(Wta - \eta_{ca} \cdot \left(\frac{\hat{Pi} \cdot Vd \cdot Ne}{Ri \cdot Ti \cdot 2}\right) + \left(\Delta Mca \cdot \frac{Ne}{2}\right)\right) + K_1 \cdot \left(Pi - \hat{Pi}\right)$$

(4.15) $$\frac{d\Delta Mca}{dt} = K_2 \cdot \left(Pi - \hat{Pi}\right)$$

## 6 Volumetric Efficiency Model-5: Variable Cam Timing Molar Volume Extension

The Andersson molar volume approach can be extended for variable cam timing by calculating the actual cylinder volumes at valve closure

(5.1)
$$Vaf = Va + Vf = Vc + Vd - Vri - \Delta Vi$$
$$Vre = Vc + \Delta Ve$$

From the engine geometrical properties and valve timing trajectories, the delta-volumes at any delta-crank position $\Delta\theta$ is

(5.2)
$$\Delta V = \left(\frac{\pi \cdot B^2}{4}\right) \cdot \left(l + a - s\right)$$
$$s = a \cdot \cos(\Delta\theta) + \left(l^2 \cdot a^2 \cdot \sin^2(\Delta\theta)\right)^{\frac{1}{2}}$$

The new residual gas mass at EVC is

(5.3)
$$Mcr = \left(\frac{Pe}{Re \cdot Te}\right) \cdot Vre$$

And the new residual gas molar volume at IVC is

(5.4)
$$Vri = \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot Vre = \left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \left(Vc + \Delta Ve\right) \cdot Fevcs + Fevco$$

The correction factors $Fevcs$ and $Fevco$ have been added to accommodate actual engine pumping associated with EVC and mitigate assumptions. The new cylinder volume available for the combined air fuel charge is

(5.5)
$$Vaf = Vc + Vd - \left(\left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \left(Vc + \Delta Ve\right) \cdot Fevcs + Fevco\right) - \left(\Delta Vi \cdot Fivcs + Fivco\right)$$

Once more correction factors $Fivcs$ and $Fivco$ for IVC has been introduced to maximize the model fidelity during the system identification from mapping datasets. Updating equation (5.12)

(5.6)
$$\eta_{ca} = \frac{Fca}{Fct(Ti,\varphi)} \cdot \left(\frac{1}{1 + \frac{\varphi}{\varphi s}}\right) \cdot \left(Vc + Vd - \left(\left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \left(Vc + \Delta Ve\right) \cdot Fevcs + Fevco\right) - \left(\Delta Vi \cdot Fivcs + Fivco\right)\right)$$

The tuning functions $Fevcs, Fevco, Fivcs$ and $Fivco$ provides means to correct for assumptions that the cylinder states of pressure and temperature equalize to the exhaust reservoir at EVC and the intake reservoir at IVC. To maximize model fidelity, these functions should be normalized around the optimal desired cam angle trajectory volumes at MBT-VCT (maximum brake torque variable cam timing) using the actual valve closure volumes and then regressed from the delta volume from the nominal angle

$$
\begin{aligned}
&Fevcs(x,\Delta Ved) = a0 + a1\cdot x + a2\cdot x^2 + a3\cdot \Delta Ved + a4\cdot \Delta Ved^2 + a5\cdot x\cdot \Delta Ved \\
&Fevco(x) = b0 + b1\cdot x + b2\cdot x^2 \\
&Fivcs(x,\Delta Vid) = c0 + c1\cdot x + c2\cdot x^2 + c3\cdot \Delta Vid + c4\cdot \Delta Vid^2 + c5\cdot x\cdot \Delta Vid \\
&Fivco(x) = d0 + d1\cdot x + d2\cdot x^2 \\
&\Delta Ved = \Delta Vevc\_mbt - \Delta Vevc\_actual \\
&\Delta Vid = \Delta Vivc\_mbt - \Delta Vivc\_actual
\end{aligned} \tag{5.7}
$$

To check the model fidelity, the slope parameters should be set to unity and the offsets to zero (no correction for assumptions). Next, model parameter sensitivity should be performed numerically from the dynamometer generated datasets. One can then derive the power regression using correlation analysis ( $Ne$ is expected to have high correlation).

## 7 Volumetric Efficiency Model-6: 10-Dimensional Energy Balance Molar Volume Extension

Starting at EVC and assuming either an isentropic or polytropic process moving the piston down with the intake valve open to the manifold reservoir, the energy balance at IVC is

(6.1) $$Ci \cdot Ti \cdot (Mca + Mcf) + Cr \cdot Te \cdot Mcr = Cc \cdot Tc \cdot (Mca + Mcf + Mcr)$$

Assuming no other energy transfer and that the specific heats $Ci = Cr = Cc$ are equal, we can solve for the IVC charge temperature:

$$Tc = \frac{Ti \cdot (Mca + Mcf) + Te \cdot Mcr}{Mca + Mcf + Mcr} = \frac{Ti \cdot \left( Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) \right) + Te \cdot Mcr}{Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) + Mcr}$$

(6.2) $$Tc = x_r \cdot Te + (1 - x_r) \cdot Ti$$

$$x_r = \frac{Mcr}{Mca + Mcf + Mcr} = \frac{Mcr}{Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) + Mcr}$$

With $x_r$ as the residual mass ratio that will change in 12 dimensional space of EGR $Ne, r_c, \varphi, Pa, Pi, Pe, Te, Tw, aCi, aCe$ and fuel stoichiometry. Using (5.5) we can calculate charge temperature in terms of the available measurements

(6.3) $$Tc = Te \cdot \left( \frac{Pi}{Pe} \right)^{\frac{\gamma - 1}{\gamma}}$$

(6.4) $$Mcr = \left( \frac{Pe}{Re \cdot Te} \right) \cdot \left( \left( \frac{Pe}{Pi} \right)^{\frac{1}{\gamma}} \cdot (Vc + \Delta Ve) \cdot Fevcs + Fevco \right)$$

$Tc$ and $Mcr$ will both be kept as state variables given their tremendous importance throughout the control strategy.

Now by utilizing **both the energy balance and molar ideal gas law** so that (7.1) can be solved for the cylinder air charge

(6.5) $$Ti \cdot \left( Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) \right) + Te \cdot Mcr = Tc \cdot \left( Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) + Mcr \right)$$

(6.6) $$Ti \cdot \left( Mca \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) \right) = Tc \cdot \left( \left( \frac{Pi \cdot Vaf}{Ri \cdot Ti} \right) \cdot \left( 1 + \frac{\varphi}{\varphi s} \right) \right) + (Tc - Te) \cdot Mcr$$

(6.7) $$Mca = \left( \frac{Tc}{Ti} \right) \cdot \left( \frac{Pi \cdot Vaf}{Ri \cdot Ti} \right) + \left( \frac{Tc - Te}{Ti} \right) \cdot \left( \frac{\varphi s}{\varphi s + \varphi} \right) \cdot Mcr$$

(6.8) $$Vaf = Vc + Vd - \left( \left( \frac{Pe}{Pi} \right)^{\frac{1}{\gamma}} \cdot Fevc \right) - Fivc$$

(6.9)
$$Fevc = \left(Vc + \Delta Ve\right) \cdot Fevcs + \left(\frac{Pe}{Pi}\right)^{\frac{-1}{\gamma}} \cdot Fevco$$

(6.10)
$$Fivc = \Delta Vi \cdot Fivcs + Fivco$$

The 10-dimensional volumetric efficiency is

(6.11)
$$\eta_{ca} = \left(\frac{Tc}{Ti}\right) \cdot \left(\frac{Vaf}{Vd}\right) + \left(\frac{Tc - Te}{Te}\right) \cdot \left(\frac{\varphi s}{\varphi s + \varphi}\right) \cdot \left(\frac{Pe}{Pi}\right) \cdot \left(\frac{Vri}{Vd}\right)$$

(6.12)
$$\eta_{ca} = \left(\frac{Tc}{Ti}\right) \cdot \left(\frac{Vc + Vd - \left(\left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot Fevc\right) - Fivc}{Vd}\right) + \left(\frac{Tc - Te}{Te}\right) \cdot \left(\frac{\varphi s}{\varphi s + \varphi}\right) \cdot \left(\frac{Pe}{Pi}\right) \cdot \left(\frac{\left(\frac{Pe}{Pi}\right)^{\frac{1}{\gamma}} \cdot \left(Vc + \Delta Ve\right) \cdot Fevcs + Fevco}{Vd}\right)$$

## *From (7.7) and (7.12) we can observe the volumetric efficiency modeling errors when we fail to include the cylinder charge temperature!*

To acquire insight into (7.7) and (7.9), it can be compared to the well established production equation (4.2)

(6.13)
$$Mca = \frac{Pi - Fpo \cdot \frac{Pa}{Pas}}{Fps} = Fms \cdot Pi + Fmo \cdot \frac{Pa}{Pas}$$

Here we can clearly see why the affine power regression correlates well by solving for air charge. A model for the residual mass is expressed in the offset

(6.14)
$$Mcr = Fmo \cdot \left(\frac{Pa}{Pas}\right) \cdot \left(\frac{Ti}{Tc - Te}\right) \cdot \left(\frac{\varphi s + \varphi}{\varphi s}\right)$$

(6.15)
$$Fmo = Mcr \cdot \left(\frac{Pas}{Pa}\right) \cdot \left(\frac{Tc - Te}{Ti}\right) \cdot \left(\frac{\varphi s}{\varphi s + \varphi}\right)$$

And the slope component relates

(6.16)
$$Fms = \left(\frac{Tc}{Ti}\right) \cdot \left(\frac{Vaf}{Ri \cdot Ti}\right)$$

Equations (7.7)-(7.11) keep the physical model **component-based** allowing integration in diverse targets based upon cost and emission objectives. Instead of utilizing sensor measurements for either $Te$ or $Pe$, other physical models can be employed without changing the state equations. However as a minimum, all development vehicles should record these parameters given the significance especially when developing PZEV-level controls (crucial during warm-up where 90% of the emissions occur in the first 60 seconds).

Numeric parameter sensitivity analysis should be performed to quantify the model parameters.

## 8 Cylinder Residual Mass Adaptive Observer

(7.1)
$$\frac{d\hat{P}i}{dt} = Kpi \cdot \left(Wt - Wc - \Delta\hat{W}cr\right) + Kypi \cdot \left(yPi - \hat{P}i\right)$$

(7.2)
$$\frac{d\Delta\hat{W}cr}{dt} = Kwcr\left(\left(\frac{Tc - Te}{Ti}\right) \cdot \left(\frac{\varphi s}{\varphi s + \varphi}\right)\right) \cdot \left(y\varphi - u\varphi\right)$$

(7.3)
$$\Delta\hat{W}cr = \frac{d\Delta\hat{W}cr}{dt} + Fwcr[n](Ne, La, \cdots)$$

$Fwcr$ is a multidimensional interpolated neural network (INN) that adapts the stationary $\Delta\hat{W}cr$ hyperspace. It is a subcomponent of a higher level parameter adaptive critique that distributes the air/fuel residuals to multiple sub-controllers. Ideally, $Fwcr$ should be adapted per cylinder if exhaust air/fuel ratio measurements can be determined cylinder-observable. Alternatively, $Fwcr$ can be derived in units of mass charge $Fmcr$ . **Proper design of the nonlinear EKF and adaptive gains are essential for high-quality performance and stability.**

**Foundation for Precision ICE Real-Time Control:**

- Dynamic and stationary cylinder charge air mass error correction across the entire operating hyperspace
- Dynamic and stationary cylinder charge temperature modeling error correction
- Dynamic and stationary cylinder residual mass modeling error correction
- Dynamic EGR feedback device for real-time adaption of production variances
- Improved AFR control (especially during start and warm-up where the charge temperature changes rapidly)
- Improved Load calculations (propagates throughout entire control strategy)
- Improved Torque calculations
- Improved NOx control (thermal miser for both SI and CI)
- Improved feedforward spark (charge temperature instead of just ECT and IAT)
- Improved ETC throttle control
- Improved engine guardian and diagnostics